ABSTRACT
Analysis of the track maintenance strategies are presented using the spot assessment of track quality (SD) and integral assessment based on the quasi cumulative distribution function (QCDF). Railway track maintenance strategy is based on analysis of the track geometry measurement, done, as a rule, with the track measuring car. The measurement data result from a random process that is fully and unambiguously described by the irregularity of the size distribution function (ISDF). Currently, SD is the digital characteristics of Track Quality Index and track maintenance planning interventional Threshold commonly used for track quality assessment, as defined by the European Railways. Properties of this function were studied that affect the accuracy of track technical condition assessment. Based on this analysis the following was found out:

i. irregularity size in track geometry measuring results does not have, as a rule, the normal distribution,
ii. track section with different extreme size irregularities may have the same SD values and vs,
iii. SD criterion does not take into account the shape of ISDF and for track section with the same SD values, contact stress and the energy dissipated in the wheel-rail contact may be different.

Therefore, using SD an interventional procedure may be not optimal.

The integral assessment is presented of track quality employing the quasi cumulative distribution function and analysis of its benefits. QCDF is the linear transformation of ISDF and returns the total length of the track irregularities with size in [mm] larger, then the interventional threshold. QCDF is invariant to the ISDF distribution form and is the monotone, continuous, and unambiguous irregularity size function. The use of the QCDF allows to identify the intervention threshold uniquely, and ensures the optimal planning and distribution of resources for track maintenance based on the objective assessment of the track technical condition.

STATE OF THE ART
Railway track maintenance strategy is based on an analysis of track geometry measurement done, as a rule, with the track measuring car. From the probability theory point of view, these measurement results are random variables that take one of the possible values with probability that depends on the track technical condition. These results are fully and unambiguously described by the irregularity size distribution function (ISDF). One of the digital characteristics of this function is the standard deviation (SD), which provides spot assessment of the track technical condition. Currently, SD is the digital characteristics of the Track Quality Index and track maintenance planning threshold as commonly used indicator for track geometry quality, as defined by the European Railways. Based on analysis of the track geometry quality deterioration process, presented in publications of (Lichberger, 2001, 2007),( Veit, 2007), the track maintenance model was proposed, which ensures obtaining a given level of the track technical condition. The model provides periodic maintenance
operations based on an analysis of the actual track condition. SD or its modifications are used as a criterion parameter value (Esveld, 1989), (Auer 2005), (Lichtberger, 2007), EN 13848-5 Standard. Based on PD CEN/TR 16513:2014 (Railway applications-Track-Survey of track geometry quality) the EN 13848-6 Standard was specified for classification of track geometry quality (Track Quality Index and Track Quality Class) across European Railway networks, in which the SD values are used as a rule. One of the justifications for the use of the SD for the track assessment quality is as follows: (British Railway Track, 1993): “Track profiles have been found to have sufficiently similar statistical properties to random processes to enable a measure of the magnitude of track irregularities to be obtained from standard deviation of the vertical and horizontal profile data.”

These publications did not demonstrate a correlation between the SD value and track technical condition, in particular its dependence on the maximum irregularity size. In addition, according to definition, SD gives only a global assessment of the track quality. As a rule, track segments with the same SD correspond to different ISDF (Krug, 2014).

As already mentioned, the ISDF is an objective and not distorted description of track geometry measurement results. In this paper we present results of the study, and SD values and ISDF used for track maintenance optimization. We present integral assessment of the track quality using the quasi cumulative distribution function. Using this function ensures the optimal planning and distribution of resources and increase the life cycle of the track.

DATABASE (informal content)
The database was developed for the relevant analysis, containing longitudinal level irregularity measurements of 1056 different quality track segments, each 200 m long; the extreme values of isolated defects have been recorded. Segments of the cumulative function F(SD) in accordance with the SD values are shown in Figure 1. The database contains the measurement results for different track sections obtained at different time. The data for each of the track segments are statistically independent, and array of the data is entirely representative.

![Figure 1 Database description](image-url)
STUDY OF PROPERTIES OF THE IRREGULARITY SIZE DISTRIBUTION FUNCTION

Examination of the Law of Distribution of Measuring Results

Study of the distribution law of measurement results is of interest for several reasons:

i. the use of the $3\sigma$ parameter to evaluate track technical condition may be justified only if the set of the measuring results corresponds to the normal distribution law,

ii. analysis of the track condition changes, checking the condition of the track according to technical requirements, as well as a comparison of the status of various track segments based on the SD value are in the terms of probability theory treated as legitimate only in cases where the measuring results correspond to the same statistical distribution law.

By default, the relevant regulations require that the distribution of the measurement results is normal. We used ANDERSON-DARLING method to test the hypothesis that the sets of measurement results follow the normal distribution law. It is one of the most powerful statistical tools for detecting most deviations from normality. Results of the analysis are presented in Figure 2 as Probability Plot.

Figure 2A Example A of Probability Plot for ANDERSON-DARLING test for two track section with the same standard deviation
Figure 2B Example B of Probability Plot for ANDERSON-DARLING test for two track section with the same standard deviation.

Probability plot was used to evaluate the fit of distribution to data estimate percentiles (probabilities) and to compare distributions of different samples. A probability plot does the following:

i. plots each value vs percentage (probability) in the sample that are less than or equal to it, along a fitted distribution line (middle blue line).

ii. may display the approximate 95% for the sample.

The curved blue lines displays the approximate 95% confidence interval for the normal distribution. Points outside the confidence intervals correspond to the tails - the extreme values of the measuring results. In accordance with the Anderson –Darling test measuring results fit normal distribution in following cases:

i. the plotted points form roughly a straight line,

ii. the plotted points fall close to the fitted distribution line,

iii. the Anderson –Darling statistic will be small, and associated p-value will be larger than chosen α-level. Statistic (p-value) is the weighed distance from the plot points to the fitted line with larger weights in the tails of the distribution. The probability plot makes it possible to perform a hypothesis test to examine whether or not the observations follow a normal distribution. For normality test, the hypotheses are:

\[ H_0: \text{data follow a normal distribution}, \]
\[ H_1: \text{data not follow a normal distribution}. \]

P-value determines the appropriateness of rejecting the null hypothesis test. The p-value is the probability of obtaining a test statistic that is at least as extreme as calculated value if the hypothesis \( H_0 \) (data follow a normal distribution) is true. The associated P-value is displayed in the plot. If \( p > (1-CI) \) \( H_0 \) is TRUE.
A commonly used CI value is 0.95. As can be seen from Figure 2A, the data sets do not satisfy these conditions. This condition does not satisfy approximately 80% of segments in the database.

**Study Of The Shape Irregularity Size Distribution Function**

The ISDF shape, in other words the ratio between the numbers of irregularities of different sizes within the track segments defines the integrated load, exerted on the track by the rolling stock. To describe the ISDF shape, application "SUM" from MINITAB software was used. This application allows obtaining a complete set of the random process parameters for detailed analysis. A sample table of the results is shown in Figure 3. This summary table contains basic statistical characteristics of the process, as well as the value of the confidence intervals for mean, median and SD.

It should be noted that specifying the limits of the confidence intervals is the necessary condition for strict analysis of each random process.

![Summary for LINE B km W,600](image1)

![Summary for LINE A km X](image2)

Figure 3 Examples A and B of the complete set measuring results for two track section with same standard deviation

Parameter which, to a certain extent, characterized the shape (relative peakedness or flatness) of the distribution function is the fourth central moment - kurtoxis-$\gamma$. Comparison of the $\gamma$ values for different track segments allows to analyze correlation between the number of irregularities of varying sizes (ISDF shape). $\gamma$ is a measure of how flat the top of symmetric distribution is when compared to a normal distribution of the same SD. $\gamma$ is actually more influenced by scores in the tails of the distribution than scores in the center of distribution.
Commonly $\gamma > 0^*$ occurs for ISDF. These distribution functions have longer tails than the normal distribution, accordingly it is often appropriate to describe this distribution as “a fat in the tails”. In this case it corresponds to the large irregularity size. At the constant SD value, an increase of $\gamma$ corresponds to an increase of scores in the tails. Figure 4 illustrates the relationship between the $\gamma$ values and ISDF form and show, as an example, parts of the ISDF for $\gamma = 12.98$ and $\gamma = 0.59$ for SD = CONST = 1.3.

![Graph of Relationship between $\gamma$ Value and ISDF](image)

Figure 4 Track segments ISDF with same SD condition comparison.

![Graph of Y Distribution](image)

Figure 5 SD- $\gamma$ two-dimensional presentation
by definition, $\Upsilon$ for normal distribution is 0, the results once again show that as a rule ISDF does not correspond to the normal distribution.

** $\Upsilon$ values depend on the global ISDF shape and therefore could not be used as a parameter of the tack geometry.

An obvious test for correlation between the values is to build diagrams in two–dimensional space. Figure 5 shows the distribution of the $\Upsilon$ base on the SD values for database segments (n=534). Shape charts are for the case of the lack of correlation between $\Upsilon$ and SD, which indicates the fact that SD does not have the ISDF shape.

**TRACK GEOMETRY MAINTENANCE PLANNING OPTIMIZATION**

In this paper we consider the optimal solution for track maintenance planning based on the results of the track geometry measurement, using as an example the measured longitudinal level irregularity size. These results can be used in the analysis of other geometric parameters of the track as well.

In our opinion, the optimization method of track maintenance should take into consideration in the planned maintenance operation only those track segments, for which the maximum irregularity size exceeding the certain value. Such value (interventional trigger) depends on the corrective maintenance policy, frequency of track inspection and defect growth rate. These values should be specified to ensure safety of operation, reduce the probability of large irregularities, and can be tightened to achieve a given level of ride comfort. In this case, it would reduce the maintenance cost and does not adversely affect the track life-cycle.

We present analysis of the track maintenance strategies using the spot assessment of the track quality based on SD and using the integral assessment based on the Quasi Cumulative Distribution Function (QCDF) which was designed for this task. For each track segment of the database, two parameters were identified that can be used to assess the track quality condition: the maximum irregularity size (MIS) and standard deviation (SD). Based on these data, for a set of segments with the same MIS values the cumulative function depending on SD value was calculated (see Figure 6).

![Cumulative functions for track sections with different max irregularity size (MIS)](image-url)
The curves in Figure 6 demonstrate the characteristic features of the ISDF properties, namely: for the same value of SD there are track segments with different MIS, for each SD value one can define a range of MIS. Almost for any SD in the list of the segments one may find segments with wide MIS range.

Figure 7 presents a solution to the inverse problem and is shown as an example of the cumulative function MIS for several SD values. The graph allows to see the problems encountered when planning works based on the SD values.

One can see that for SD>2mm (n=357) MIS is 15 mm and 40% of the track segments have the MIS value less than 11mm. When choosing the SD values within 1.4-2.4 mm (n=443) as EN 13848-5 recommends, MIS is 11mm and for 45% of the track segments MIS is smaller than 8 mm.

![Figure 7 MIS cumulative function for different SD values](image)

As mentioned above, ISDF gives a full and distorted description track measurement results. Naturally, any linear transformation of this function also allows to get distorted information about this process. We used a linear transformation of this function that describes the accumulated length \( \ell_\Sigma \) of the various irregularity sizes.

We call this function the Quasi Cumulative Distribution Function (QCDF), because, unlike classical cumulative function, QCDF returns the absolute value of the irregularity accumulated length \( \ell_\Sigma \). The concept of the structure of this function for track quality assessment is shown in Figure 8.
There are two curves in the chart. The first one is the original ISDF curve. It is a function of the distribution which shows the length of each size of irregularity for the selected track segment. In essence, it is a histogram of values obtained from the measuring results set. The second curve shows the accumulated length of the track irregularity with size equal or larger than the threshold $S_s$ specified for the analysis $\ell_\sum s^4$. For example, as can be seen from graph, the accumulated length of the irregularities $\ell_\sum 4$ with size equal and more than 4 mm is 16m.

QCDF properties:

- $0 \leq QCDF(s) \leq L$ - QCDF value can vary from 0 to the track segment length,
- $QCDF(0)=L$ - for $s=0$ accumulated length equal to the length of the track segment,
- $QCDF(\infty)=0$ - the accumulated length for irregularity value $\infty$ is 0,
- if $s_1 < s_2$, then $QCDF(s_1)>QCDF(s_2)$ - the accumulated length is monotonically decreasing.

The values of the QCDF depend only on the values of the ISDF, but not on the distribution type.

The software was developed to use QCDF for optimization of track maintenance that allows selecting track segments with the preset $S_{max}$ from list. Based on the QCDF analysis for these track segments, including the accumulated lengths of varying sizes of irregularities and other factors (track deterioration, etc.) one can make optimal decision regarding the work planning.

To illustrate the use of QCDF for optimizing track maintenance, Figure 9a shows the set of QCDF for track segments with $S_{max}>11$ mm and Figure 9b shows the set of QCDF for track segments with $SD=CONST =1.56$ mm.
Figure 9a QCDF for track segments with $S_{\text{MAX}} \geq 11\text{mm}$

Figure 9b QCDF for track segments with $\text{SD} = 1.56\text{mm} = \text{CONST}$
CONCLUSIONS

i. Measurement results of the track irregularities size (ISDF) do not follow, as a rule, the normal distribution, because comparison between the parameters of various statistical distribution laws, which are calculated by different formulas, do not prove it.

ii. Track segments with different $S_{\text{max}}$ may have same SD values and vice versa.

iii. SD criterion does not indicate the shape of ISDF and for track section with the same values of SD and with different contact stress and energy dissipated in the wheel – rail contact will be different.

iv. Each SD value corresponds with track segments with different $S_{\text{max}}$, so using SD does not provide the optimal maintenance planning; the SD criterion may correspond to segments of varying irregularity sizes which leads to the unnecessary additional cost and reduction of track service life.

v. Optimal solutions for planning of track maintenance can be obtained by using QCDF that has been developed on purpose for maintenance planning task. QCDF returns the integral assessment of the track quality in the form of irregular dimensions and accumulated length and allows to select track segments with given parameters.

vi. Developed software for QCDF calculation and analysis can be used with any type of measuring equipment.

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